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TEMPERATURE DISTRIBUTIONS IN COUETTE FLOW WITH AND WITHOUT ADDITIONAL PRESSURE GRADIENT

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Abstract—Temperature distributions in constant viscosity, incompressible Couette-type of fluid flow at constant pressure and under an additional pressure gradient were analysed. Equations for temperature distributions are derived.

Also numerical results are given for the constant pressure Couette flow (linear velocity profile) and three flow situations in which Couette flow is assisted by pressure gradients of increasing magnitude (parabolic velocity profiles). Graphs relating local Nusselt number and bulk temperature (cup'mixing temperature) to longitudinal distance from entry are given.

NOMENCLATURE

$Ai(x)$, $Bi(x)$, Airy functions of argument x;
C_p, heat capacity at constant pressure $[L^2t^{-2}T^{-1}]$;
$C(\lambda_n, \beta)$, $D(\lambda_n, \beta)$, coefficients in infinite series expansion (equation 19), defined in equations (24) and (25);
$f(\lambda_n, \beta, \xi)$, $g(\lambda_n, \beta, \xi)$, eigenfunctions defined in equation (23);
h, heat-transfer coefficient $[Mt^{-3}T^{-1}]$;
k, thermal conductivity $[MLt^{-3}T^{-1}]$;
$K(\lambda_n, \beta, \xi)$, linear combination of eigenfunctions f and g, equation (22);
$M(a, b, x)$, confluent hypergeometric function of argument x and parameters a and b;
P, pressure $[ML^{-1}t^{-2}]$;
$p(\xi, \mu_n)$, $r(\xi, \mu_n)$, eigenfunctions defined in equation (14);
s, Laplace-transformation variable;
t, time $[t]$;
T, temperature $[T]$;
v_x, fluid velocity in x-direction $[Lt^{-1}]$;
V_0, fluid velocity at $y = \delta$ $[Lt^{-1}]$;
w, transformed coordinate in y-direction defined by (17);
x, direction of flow $[L]$;
y, direction perpendicular to flow $[L]$;
z, transformed coordinate in x-direction defined by (17);
α, thermal diffusivity $[L^2t^{-1}]$;
γ, dummy integration variable, equation (13);
$\Gamma(x)$, gamma function of argument x;
δ, distance between the two plates confining the fluid flow;
η, dimensionless distance in x-direction $(x/\delta)\,Pe^{-1}$;
θ, dimensionless temperature, $(T - T_0)/(T_2 - T_0)$;
λ, eigenvalue, equation (20);
μ, eigenvalue, equation (15) or viscosity $[ML^{-1}t^{-1}]$;

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ξ, dimensionless distance in y-direction, y/δ;
ρ, fluid density, $[ML^{-3}]$.

Dimensionless criteria
Pe_1, Péclét number $(= \langle v_x \rangle \delta/\alpha)$;
Pe, Péclét number $(= V_0 \delta/\alpha)$;
Nu_{loc}, local Nusselt number$(= h_{loc}\delta/k)$.

Superscripts
∞, at very large x-coordinate;
$-$, Laplace transformed quantity.

Subscripts
0, at entrance $(x = 0)$;
1, at $y = 0$;
2, at $y = \delta$;
n, rank number of eigen value;
t, transient;
m, cup mixing;
loc. local.

1. INTRODUCTION

HEAT transfer to liquids in Couette flow with pressure gradients in the flow direction is of interest in, e.g. the study of thermal pasteurization processing of liquid foods, heat transport in bearings (Vogelpohl [1]) and transport phenomena in falling-film flow with surface tension gradients at the free surface (Bird *et al.* [2], p. 66).

In Fig. 1 the flow situation is sketched. The liquid has a velocity V_0 at $y = \delta$ (e.g. by the movement of a cylindrical surface or induced by a surface tension gradient in x-direction). The flow in x-direction is assisted by a pressure gradient $(-dP/dx)$.

Several investigators reported analyses of temperature distributions in Couette-type of liquid flows, e.g. Vogelpohl [1], Pai [3], Bhatnagar and Tikekar [4], and Purohit [5]. In these reports a number of physically different flow conditions and boundary conditions have been considered. Vogelpohl [1], for instance, calculated the temperature profiles generated by viscous dissipation of energy. He analysed the stationary situation without pressure gradient ($\beta \to \infty$ in Fig. 1) extensively, including the temperature profiles in the entrance region where the temperature depends on both y and x coordinates and the variable viscosity. The flow situation with pressure gradient, $\beta < \infty$ in Fig. 1, was analysed briefly, giving a solution for temperature profiles arising from dissipation of viscous energy valid in the limit $x \to \infty$. Pai [3] and Bhatnagar [4] discussed various situations of instationary heat generation by some source (not viscous dissipation) in Couette flow between concentric cylindrical surfaces. In these studies the temperature was assumed to be independent of the x coordinate (see Fig. 1).

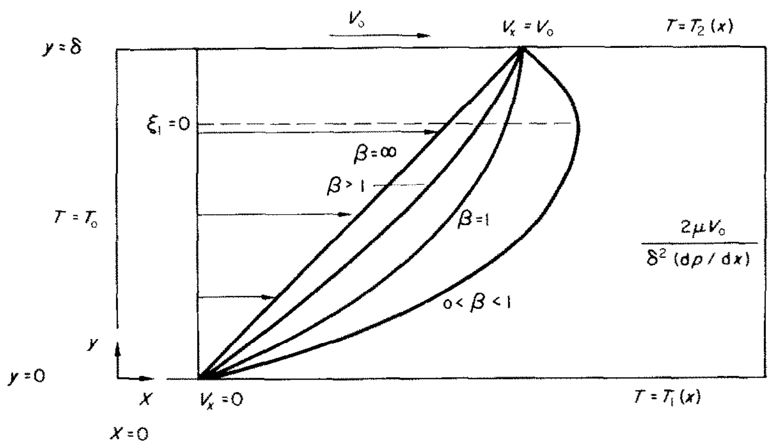

FIG. 1. Schematical picture of Couette-flow situation with and without pressure gradients.

Purohit [5] extended the solutions obtained by Bhatnager to include viscous dissipation of energy and time dependent temperature of one of the cylindrical surfaces. In all these studies the pressure was assumed to be uniform.

In the present work solutions are given for developing temperature profiles in Couette flow with and without an additional pressure gradient. The two plates have different temperatures, T_1 and T_2, which may be a function of x for the constant pressure flow situation. This problem is of concern at this laboratory in screening of different flow systems for continuous thermal pasteurization processing of food liquids. The death rate of microorganisms can be considered as a first order chemical reaction, where the reaction constant is strongly temperature dependent. The objective of pasteurization is to reduce the concentration of certain microorganisms to a level that insures extended storage life, safety to the consumer, or both. Not infrequently, the heat treatment required for adequate pasteurization causes deleterious changes in odor, flavor, or functional properties of the food. The rates of the two processes (i.e. the killing of microorganisms and the changes in properties of the food) are not necessarily of the same order, nor do they exhibit the same temperature dependence.

In general, pasteurization systems are designed to subject each particle of liquid to some stipulated minimum heat-treatment. Where the time-temperature is non-uniform, most of the fluid must receive substantially more than the required minimum dosage. Thus the fluid as a whole receives more heat treatment than is theoretically necessary for adequate pasteurization, at the risk of lowered quality. Flow systems as indicated in Fig. 1 have potentials in providing a more uniform time-temperature treatment to food liquids than that one can achieve in tube flow.

2. THE MODEL

The energy equation for fluids of constant density, viscosity and heat conductivity (see Bird *et al.* [6], p. 324) can be simplified to:

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x}\right) = k\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right). \qquad (1)$$

Schlichting, [7], p. 78, gives the following relation for v_x:

$$v_x = \left(\frac{y}{\delta}\right)V_0 - \frac{\delta^2}{2\mu}\frac{\mathrm{d}P}{\mathrm{d}x}\left(\frac{y}{\delta}\right)\left(1 - \frac{y}{\delta}\right). \qquad (2)$$

As is usually done one can safely neglect the diffusion of heat in the flow direction compared with the convective transport of heat in this direction ($\alpha\partial^2T/\partial x^2 \ll v_x\partial T/\partial x$). Also steady state will be assumed. Introducing dimensionless variables and substituting equation (2) into equation (1) gives the following differential equation:

$$\xi\{1 + \beta^{-1}(1 - \xi)\}\frac{\partial T}{\partial \eta} = \frac{\partial^2 T}{\partial \xi^2}, \qquad (3)$$

where

$$\xi = y/\delta \qquad (4)$$

$$\beta = -\frac{2\mu V_0}{\delta^2(\mathrm{d}P/\mathrm{d}x)} \qquad (5)$$

$$\eta = \frac{\alpha x}{V_0\delta^2} = \left(\frac{x}{\delta}\right)Pe^{-1}. \qquad (6)$$

If the parameter $\beta = \infty$, pure Couette flows without pressure gradient is referred to. If $\beta^{-1} > 0$ the fluid flow is assisted by a negative pressure gradient in the x-coordinate direction.

The following boundary conditions to equation (3) are chosen:

$$\left.\begin{array}{llll} \text{(i)} & \xi = 0, & \eta > 0, & T = T_1(\eta), \\ \textit{(ii)} & \xi = 1, & \eta > 0, & T = T_2(\eta), \\ \text{(iii)} & \eta = 0, & 0 \leqslant \xi \leqslant 1, & T = T_0. \end{array}\right\} \qquad (7)$$

The fluid enters the system at a uniform temperature T_0 and is in contact with two walls at different temperatures, T_2 and T_1. Both T_2 and T_1 are allowed to vary with x.

In the next two sections the solution of equation (3) with boundary conditions (7) will be discussed. First a solution for flow at constant pressure ($\beta = \infty$) will be given, subsequently a solution for Couette flow with pressure gradient will be given.

3. FLOW AT CONSTANT PRESSURE ($\beta = \infty$)

Applying Laplace transformation, the transform being defined by

$$\mathscr{L}\{T\} \equiv \bar{T} = \int_0^\infty T \exp(-s\eta)\,\mathrm{d}\eta, \qquad (8)$$

to equations (3) and (7) gives the differential equation

$$\frac{\mathrm{d}^2\bar{T}}{\mathrm{d}\xi^2} - s\xi\bar{T} = 0, \qquad (9)$$

with boundary conditions:

$$\bar{T} = \bar{T}_1, \quad \xi = 0, \quad \eta > 0, \qquad (10)$$

$$\bar{T} = \bar{T}_2, \quad \xi = 1, \quad \eta > 0. \qquad (11)$$

The solution of (9) can be conveniently stated using Airy functions, see Abramowitz and Stegun [8], p. 446.

The solution is:

$$\bar{T} = \bar{T}_1 \frac{Ai(\sqrt[3]{s})\,Bi(\xi\sqrt[3]{s}) - Bi(\sqrt[3]{s})\,Ai(\xi\sqrt[3]{s})}{Ai(0)\,[(\sqrt{3})\,Ai(\sqrt[3]{s}) - Bi(\sqrt[3]{s})]} + \bar{T}_2 \frac{(\sqrt{3})\,Ai(\xi\sqrt[3]{s}) - Bi(\xi\sqrt[3]{s})}{(\sqrt{3})\,Ai(\sqrt[3]{s}) - Bi(\sqrt[3]{s})}. \qquad (12)$$

Inversion from the s-domain to the η-domain by using the inversion integral and the convolution theorem gives:

$$T = 3^{\frac{2}{3}}\,\Gamma(\tfrac{2}{3}) \sum_{n=1}^{\infty} \frac{p(\xi,\mu_n)}{q^1(\mu_n)} \int_0^\eta T_1(\eta-\gamma)\exp(-\mu_n\gamma)\,\mathrm{d}\gamma + \sum_{n=1}^{\infty} \frac{r(\xi,\mu_n)}{q^1(\mu_n)} \int_0^\eta T_2(\eta-\gamma)\exp(-\mu_n\gamma)\,\mathrm{d}\gamma, \qquad (13)$$

where $p(\xi, \mu_n)$, $r(\xi, \mu_n)$ and $q^1(\mu_n)$ are defined by:

$$p(\xi,\mu_n) = Ai(-\sqrt[3]{\mu_n})\,Bi(-\xi\sqrt[3]{\mu_n}) - Bi(-\xi\sqrt[3]{\mu_n})\,Ai(-\xi\sqrt[3]{\mu_n}),$$

$$r(\xi,\mu_n) = (\sqrt{3})\,Ai(-\xi\sqrt[3]{\mu_n}) - Bi(-\xi\sqrt[3]{\mu_n}), \qquad (14)$$

$$q^1(\mu_n) = \tfrac{1}{3}\mu_n^{-\frac{2}{3}}\{(\sqrt{3})\,Ai^1(-\sqrt[3]{\mu_n}) - Bi^1(-\sqrt[3]{\mu_n})\}.$$

In equations (13) and (14) μ_n are the roots of the characteristic equation

$$(\sqrt{3})\,Ai(\sqrt[3]{\mu_n}) - Bi(\sqrt[3]{\mu_n}) = 0. \qquad (15)$$

The equations (13)–(15) can also be derived in a notation using Besselfunctions of order $-\frac{1}{3}$ and $\frac{1}{3}$. It was decided to use the Airy functions because they are slightly easier to compute numerically.

If the temperatures T_1 and T_2 are constant the solution for T simplifies to:

$$\theta = \frac{T - T_0}{T_2 - T_0} = (1-\xi) + 3^{\frac{2}{3}}\,\Gamma(\tfrac{2}{3}) \sum_{n=1}^{\infty} \frac{p(\xi,\mu_n)}{\mu_n q^1(\mu_n)} \exp(-\mu_n\eta) + \frac{T_1 - T_0}{T_2 - T_0} \left[\xi + \sum_{n=1}^{\infty} \frac{r(\xi,\mu_n)}{\mu_n q^1(\mu_n)} \exp(-\mu_n\eta)\right]. \qquad (16)$$

In Fig. 2 the first five eigen functions $r(\xi, \mu_n)$ are given as a function of ξ. Figure 3 gives temperature profiles computed with equation (16) for the case $(T_1 - T_0)/(T_2 - T_0) = 0{\cdot}5$.

4. FLOW WITH PRESSURE GRADIENT ($\beta^{-1} > 0$)

If a pressure gradient is applied to push the fluid through the slit in the x-direction two different cases are possible. If $\beta \geqslant 1$, the velocity profile will show no maximum in the space between the two plates, the velocity has a maximum at $\xi = 1$.

If the pressure gradient is increased further, a maximum in the velocity profile appears. Both these cases will be covered.

In the limit $\beta \to 0$ (or $V_0 = 0$) the system reduces to laminar flow between two flat plates. This situation has been treated extensively in a number of papers: Prins, Mulder and Schenk [9]; Van der Does de Bye and Schenk [10]; Schenk [11]; Schenk and Beckers [12]; Schenk and Dumoré [13]; Lauwerier [14] and later by Sellars *et al.* [15].

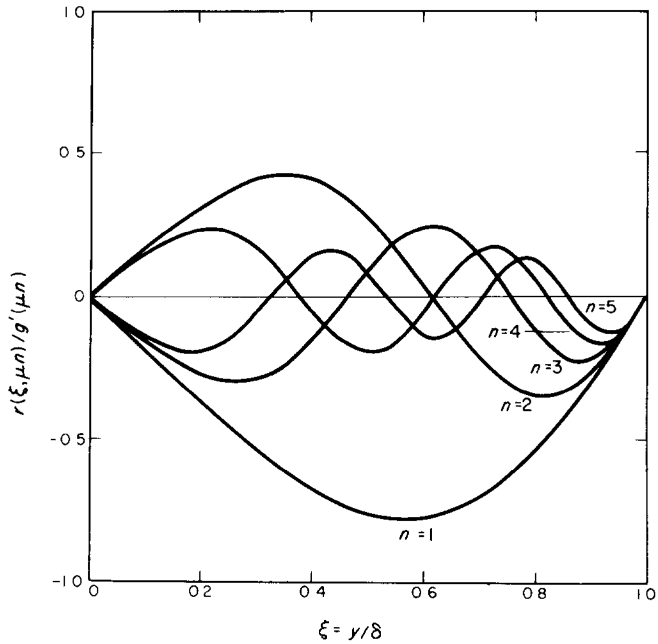

FIG. 2. First five eigen functions $r(\xi, \mu_n)$ of equation (16).

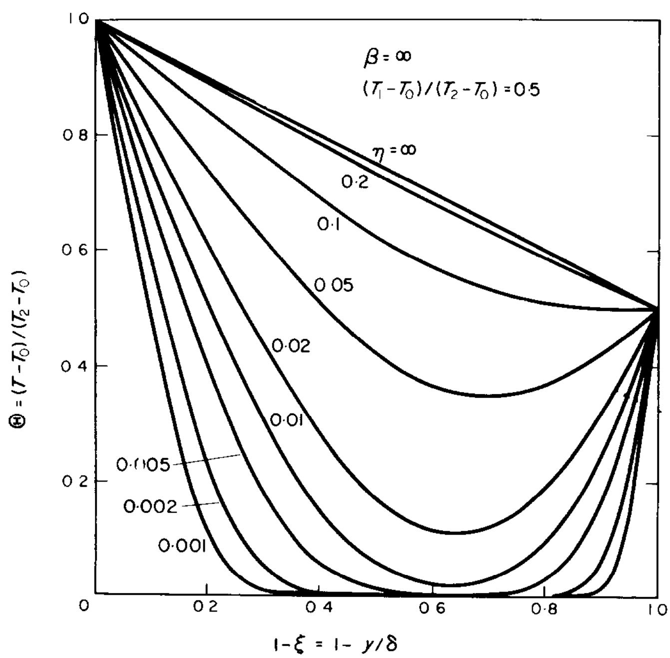

FIG. 3. Temperature distributions in constant pressure Couette-flow, $(T_1 - T_0)/(T_2 - T_0) = 0{\cdot}5$.

Using a coordinate transformation

$$w \equiv \tfrac{1}{2}(1+\beta) - \xi;$$

$$z \equiv \eta\beta = \left(\frac{1}{2\beta} + \frac{1}{6}\right)\left(\frac{x}{\delta}\right) Pe_1^{-1}. \qquad (17)$$

Equation (3) becomes:

$$[\tfrac{1}{4}(1+\beta)^2 - w^2]\frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial w^2}. \qquad (18)$$

The physical meaning of this transformation is that the point $w = 0$ coincides with the maximum in the velocity profile. As a result, the differential equation takes a form which has confluent hypergeometric functions as its solutions (Lauwerier [14]).

Separation of variables in equation (18) gives the solution:

$$\theta = (1-\xi) + \sum_{n=0}^{\infty} C(\lambda_n,\beta) K(\lambda_n,\beta,\xi)\exp(-\lambda_n^2\eta\beta) + \frac{T_1 - T_0}{T_2 - T_0} \times [\xi + \sum_{n=0}^{\infty} D(\lambda_n,\beta) K(\lambda_n,\beta,\xi)\exp(-\lambda_n^2\eta\beta)]. \qquad (19)$$

The λ_n are positive roots of the following characteristic equation:

$$g(\lambda_n,\beta,0) f(\lambda_n,\beta,1) - g(\lambda_n,\beta,1) f(\lambda_n,\beta,0) = 0. \qquad (20)$$

The functions $K(\lambda_n,\beta,\xi)$ are solutions of the Sturm–Liouvillian system:

$$K'' + \lambda^2[\tfrac{1}{4}(1+\beta)^2 - \xi_1^2] = 0$$
$$K(\lambda,\beta,\tfrac{1}{2}(\beta+1)) = K(\lambda,\beta,\tfrac{1}{2}(\beta-1)) = 0. \qquad (21)$$

The function $K(\lambda_n,\beta,\xi)$ is given by:

$$K(\lambda_n,\beta,\xi) = g(\lambda_n,\beta,\xi) f(\lambda_n,\beta,0) - g(\lambda_n,\beta,0) f(\lambda_n,\beta,\xi) \qquad (22)$$

while $f(\lambda,\beta,\xi)$ and $g(\lambda,\beta,\xi)$ are defined as follows:

$$f(\lambda,\beta,\xi) = \exp[-\tfrac{1}{2}\lambda\{\tfrac{1}{2}(1+\beta) - \xi\}^2] \times M[\tfrac{1}{4} - \tfrac{1}{16}\lambda(1+\beta)^2, \tfrac{1}{2}, \lambda\{\tfrac{1}{2}(1+\beta) - \xi\}^2], \qquad (23a)$$

and

$$g(\lambda,\beta,\xi) = (\sqrt{2\lambda})[\tfrac{1}{2}(1+\beta) - \xi] \exp[-\tfrac{1}{2}\lambda\{\tfrac{1}{2}(1+\beta) - \xi\}^2] \times M[\tfrac{3}{4} - \tfrac{1}{16}\lambda(1+\beta)^2, \tfrac{3}{2}, \lambda\{\tfrac{1}{2}(1+\beta) - \xi\}^2]. \qquad (23b)$$

The constants C and D in equation (19) follow from

$$C(\lambda_n,\beta) = \frac{\int_0^1 (1-\xi)[\tfrac{1}{4}(1+\beta)^2 - \xi^2] K(\lambda_n,\beta,\xi)\,d\xi}{\int_0^1 [\tfrac{1}{4}(1+\beta)^2 - \xi^2] K^2(\lambda_n,\beta,\xi)\,d\xi}, \qquad (24)$$

and

$$D(\lambda_n,\beta) = -\frac{\int_0^1 \xi[\tfrac{1}{4}(1+\beta)^2 - \xi^2] K(\lambda_n,\beta,\xi)\,d\xi}{\int_0^1 [\tfrac{1}{4}(1+\beta)^2 - \xi^2] K^2(\lambda_n,\beta,\xi)\,d\xi}. \qquad (25)$$

In Fig. 4 the behavior of the characteristic equation (20) is sketched as a function of λ for a number of values of β. For $\beta = 1$ the eigen values coincide with those obtained in the

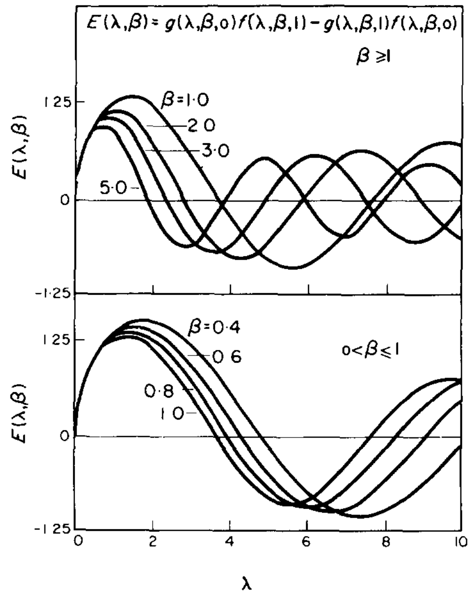

FIG. 4. Characteristic equation for Couette-flow with additional pressure gradient.

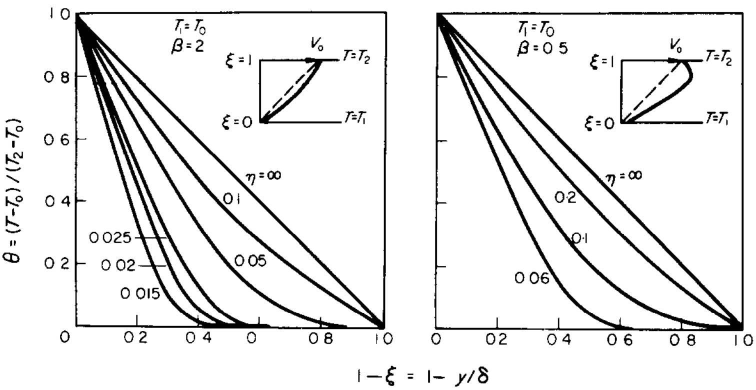

FIG. 5. Temperature distributions in Couette-flow with additional pressure gradient.

analysis of heat transfer in a centrifugal film evaporator, Bruin [16]. And, as shown in [16], one can derive the following asymptotic equation for λ_n, valid for large n ($\geqslant 4$):

$$\lambda_n = 4n + \tfrac{11}{3} \quad (\beta = 1). \tag{26}$$

The coefficients $C(\lambda_n, \beta)$ and $D(\lambda_n, \beta)$ were calculated using a numerical integration procedure (Simpson-rule) to evaluate numerator and denominator. The calculations were performed for three values of β(0·5; 1·0; 2·0) and for 4 eigen values λ_n. For larger values of n computational difficulties were encountered in determination of eigen values λ_n and the expansion coefficients $C(\lambda_n, \beta)$ and $D(\lambda_n, \beta)$. In Fig. 5 temperature profiles are given for different values of η at $\beta = 0{\cdot}5$ and $\beta = 2{\cdot}0$. The velocity profiles to which these two β-values pertain are indicated.

Local Nusselt numbers and average temperatures

The local Nusselt number (Nu_{loc}) at the moving plate ($y = \delta$) is defined by:

$$Nu_{\text{loc}} = \frac{h_{\text{loc}}\delta}{k} = -\frac{1}{1-\theta_m}\left(\frac{\partial\theta}{\partial\xi}\right)\bigg|_{\xi=1} \tag{27}$$

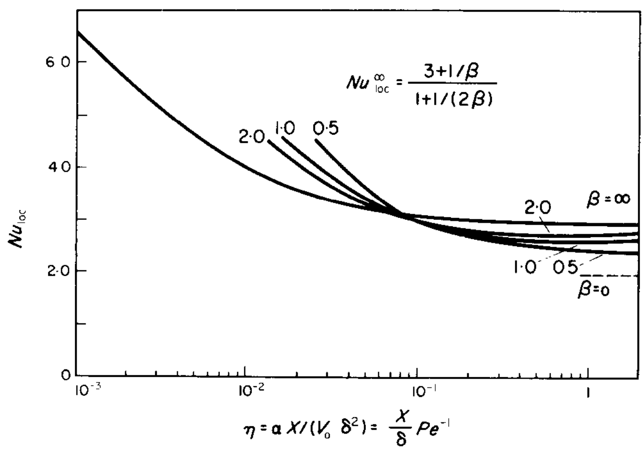

FIG. 6. Local Nusselt number as a function of x-coordinate; β as parameter.

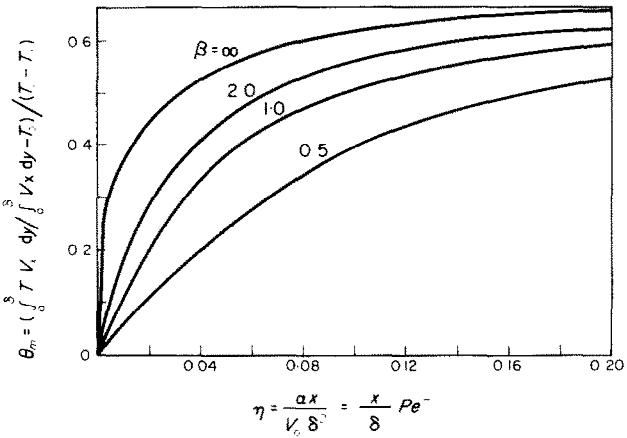

FIG. 7. Cup mixing temperature as a function of x-coordinate; β as parameter.

where θ_m is taken as the cup-mixing temperature of the flowing liquid:

$$\theta_m = \frac{\int_0^1 v_x \theta \, d\xi}{\int_0^1 v_x \, d\xi}. \tag{28}$$

The limiting value for the Nusselt number at large distances downstream, were the temperature profile has become linear, can be found from (2), (27) and (28):

$$Nu_{loc}^{\infty} = \frac{1}{1 - \theta_m} = \frac{3 + 1/\beta}{1 + 1/(2\beta)}. \tag{29}$$

In Fig. 6 the dependence of Nu_{loc} on the x-coordinate is given for the case $T_1 = T_0$ (one-sided heat penetration into the flowing liquid) and for different values of β. Figure 7 presents the cup-mixing temperature (θ_m) as a function of the x coordinate for different values of β.

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DISTRIBUTION DE TEMPÉRATURE POUR UN ÉCOULEMENT DE COUETTE AVEC ET SANS GRADIENT DE PRESSION

Résumé—On a analysé des distributions de température dans un écoulement de type Couette pour un fluide incompressible à viscosité constante, à pression constante et sous un gradient de pression. Des équations pour les distributions de température en sont dérivées.

On donne aussi des résultats numériques pour l'écoulement de Couette à pression constante (profil de vitesse linéaire) et trois situations d'écoulement dans lesquelles l'écoulement de Couette est accompagné de gradients de pression de grandeur croissante (profils de vitesse paraboliques). On donne des graphes représentatifs du nombre de Nusselt local et de la température de mélange en fonction de la distance longitudinale à partir de l'entrée.

DIE TEMPERATURVERTEILUNG IN EINER COUETTE-STRÖMUNG MIT UND OHNE ZUSÄTZLICHEN DRUCKGRADIENTEN

Zusammenfassung—Es wurde die Temperaturverteilung in einer inkompressiblen Flüssigkeitsströmung vom Couette-Typ bei konstanter Viskosität und konstantem Druck mit einem zusätzlichen Druckgradienten untersucht. Die Gleichungen für die Temperaturverteilungen werden abgeleitet.

Numerische Ergebnisse werden für die Couette-Strömung unter konstantem Druck (lineares Geschwindigkeitsprofil) angegeben und für drei Fälle, bei denen die Couette-Strömung mit Druckgradienten wachsender Grösse verbunden ist (parabolische Geschwindigkeitsprofile). Graphisch wird die lokale Nusselt-Zahl und die Mischtemperatur in Abhängigkeit von der Längsentfernung vom Eintritt dargestellt.

РАСПРЕДЕЛЕНИЯ ТЕМПЕРАТУРЫ В ПОТОКЕ КУЭТТА ПРИ НАЛИЧИИ И ОТСУТСТВИИ ДОПОЛНИТЕЛЬНОГО ГРАДИЕНТА ДАВЛЕНИЯ

Аннотация—Проведен анализ распределений температуры в несжимаемом с постоянной вязкостью потоке жидкости типа потока Куэтта при постоянном давлении и при наличии дополнительного градиента давления. Выведены уравнения распределения температуры.

Приведены также численные результаты для течения Куэтта с постоянным давлением (линейный профиль скорости) и три случая наличия при течении Куэтта градиентов давления возрастающего значения (параболические профили скорости). Приведены графики зависимостей локальных чисел Нуссельта и объемной (среднесмешанной) температуры от расстояния до входа.